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TITLE- Spin-up of Skylab B for Artificial Gravity

TM- 70-1022-14

DATE- September 30, 1970

FILING CASE NO(S)- 620

AUTHOR(S)-L. E. Voelker

FILING SUBJECT(S) (ASSIGNED BY AUTHOR(S))- Skylab Program
Artificial Gravity
CSM Guidance and Control
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ABSTRACT

The Service Module Reaction Control System (SM-RCS) is capable of performing the spin-up maneuver of Skylab B for an artificial gravity experiment. Since constant spin is stable only about the axis of maximum moment of inertia, the spin-up torque should be applied about this axis. The RCS thrusters can apply an average torque about this axis by operating in a pulsed mode. Using a minimum fuel open-loop procedure for applying a specified average torque, the rigid body dynamics are simulated during and after spin-up.

Spin-up to four rpm producures negligible wobble when the location of the axis of maximum moment of inertia is known precisely. If there is a substantial error in the knowledge of the location of this axis, the same spin-up procedure produces larger, but acceptable amplitudes of wobble. These judgements are based on preliminary estimates, based on ground test data, of the physiological threshold for wobble detection by the semicircular canals of the inner ear.

The rotation of the spin axis from the solar vector due to the pulsed thruster procedure is small, and results in a negligible electrical power penalty.

BA-145A (8-68)

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BA-146 (8-64)

BELLCOMM, INC.
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SUBJECT: Spin-up of Skylab B for Artificial Gravity - Case 620

DATE: September 30, 1970

FROM: L. E. Voelker

TM 70-1022-14

TECHNICAL MEMORANDUM

1.0 INTRODUCTION

An artificial gravity experiment has been proposed for the Skylab B mission. Spinning the Skylab produces a centrifugal force field and the artificial gravity environment. Assuming a torque-free external environment, stable constant spin can exist only about the axis of maximum principal moment of inertia. Ideally, constant spin would be achieved by applying a torque only about this principal axis. The Service Module Reaction Control System, which is the only Skylab system capable of executing the spin-up maneuver, cannot continuously apply such an ideal torque. However, the correct average torque can be applied by the RCS thrusters by operating them in a pulsed mode.

This report gives the results of an analysis of the rigid body dynamics of Skylab B during spin-up by an open-loop, pulsed thruster procedure that is optimized for minimum fuel consumption. Determination of the thruster firing profile requires prior knowledge of the location of the principal inertia axes system relative to body coordinates, and the effects of miscalculation of this location on the vehicle dynamics are included.

2.0 ANALYSIS

2.1 Inertia Properties

Skylab B inertia properties were derived by increasing Skylab A consumable supplies to a nine-month mission capability and adding two ballast masses on deployable booms as shown in Figure 1. These 10 slug masses located at (X_{cg}, 100, -10) feet, and at (X_{cg}, -100, 10) feet in Skylab coordinates, reduce the angle between the axis of maximum principal moment of inertia and the Skylab Z-axis (solar array normal) to approximately 4° (Reference 1). Close alignment of these axes is required to achieve high solar array power output when spinning about the solar vector. The 4° is caused by the offset ATM and it is impractical to attempt to reduce it further. Unit vectors along the axes of maximum, intermediate, and minimum principal moments of inertia are defined as $\underline{U}_{MAX}$, $\underline{U}_{INT}$, $\underline{U}_{MIN}$, respectively.

In Reference 1 it was shown that if the ballast booms were both shortened by 30 feet to 70 feet in length, the angle between the Skylab Z-axis and $\underline{U}_{MAX}$ would increase by only one degree. This increase results from an approximate 3° rotation of $\underline{U}_{MAX}$ about $\underline{U}_{MIN}$. The 100 foot booms were chosen for the stability in inertia axis location that they offer. If the booms are shortened below 70 feet, a small change in length causes a large rotation of $\underline{U}_{MAX}$ about $\underline{U}_{MIN}$.

There will be some uncertainty in the knowledge of the location of $\underline{U}_{MAX}$ in Skylab coordinates, however. This uncertainty is expressed as an error angle δ measured from the assumed location to the actual location. With such an angular error, the spin-up procedure will be applying the average torque about the wrong axis. The error angle δ is given the following values in this analysis to determine its effect on vehicle dynamics: 0°, implying perfect knowledge of the inertia properties; ±5° in the plane of $\underline{U}_{MAX}$ - $\underline{U}_{INT}$: and ±0.5° in the plane of $\underline{U}_{MAX}$ - $\underline{U}_{MIN}$. An error of 5° about $\underline{U}_{MIN}$ in the knowledge of $\underline{U}_{MAX}$ implies an error in the product of inertia I_{yz} of approximately 15,000 slug-ft^2. For I_{yz} to be in error by this magnitude, the equivalent of 5000 pounds would have to be mislocated by 14 feet on the circumference of the Workshop. An error of 0.5° about $\underline{U}_{INT}$ in the knowledge of $\underline{U}_{MAX}$ implies an error in the product of inertia I_{xz} of approximately 30,000 slug-ft^2. This error in I_{xz} could result from a mislocation of 2500 pounds from one side of the Workshop to the other side 20 feet aft of the Skylab center of mass. The above values of δ are thus considered to be very conservative bounds on the error angle.

2.2 Pulsed Thruster Spin-up

The pulsed thruster spin-up procedure used in this study is based on a minimum fuel solution for applying specified torque to the Skylab with the SM RCS that was reported in Reference 2. The procedure employs three of the 16 thrusters. Each thruster is fired for a specific portion of a basic period of ten seconds, which is repeated for eight minutes to attain a spin rate of 4 rpm.* At this spin rate, the centrifugal acceleration at the Workshop floor is 0.18 G, slightly more than lunar gravity. The ten-second period in the thruster firing sequence is arbitrary and was chosen to be long enough so that the thrusters would develop maximum specific impulse but not so long that large angular velocity components about axes other than $\underline{U}_{MAX}$ would develop.

A constant torque about $\underline{U}_{MAX}$ of approximately 4000 foot-pounds would accelerate Skylab B at the specified 0.5 RPM/minute.* Put another way, an angular impulse of 40,000 foot-pound-seconds is required over each 10 second period. A linear impulse acting in the plane of the RCS quads perpendicular to $\underline{U}_{MAX}$ and through the point P, which marks the intersection of $\underline{U}_{MIN}$ with the RCS plane, will provide an angular impulse of the correct sense. This linear impulse vector, $\bar{I}$, is shown in Figure 2. The proper magnitude of $\bar{I}$ is calculated by dividing the 40,000 ft. lb. sec. angular impulse by the distance between $\bar{I}$ and the Skylab center of mass, which is about 42 feet.

With the data on required magnitude and location of $\bar{I}$ in the plane of the RCS quads, a minimum fuel solution for the thruster firing times, within the 10 second period, can be obtained directly from Reference 2. The solution is such that the sum of the impulses from the three thrusters used matches $\bar{I}$ in magnitude, direction, and line of action. The lengths of the pulses for the case at hand are shown in Figure 2 for one basic 10 second period. Figure 2 also shows the profile of the moment components about the principal axes of inertia. The time integrals of the moment components over the basic period give identically the proper impulse components. That is,

$$\int_0^{10} M_{MIN}dt = \int_0^{10} M_{INT}dt = 0, \text{ and } \int_0^{10} M_{MAX}dt = 40{,}000 \text{ ft-lb-sec.}$$

The sequence of the thruster firings in the basic period was selected to minimize the integrals of the absolute value of the moments about the minimum and intermediate principal axis over the period.

Although the ten-second integral of the applied moment about $\underline{U}_{MIN}$ and $\underline{U}_{INT}$ is zero, we note from Figure 2 that the instantaneous moment about these axes is non-zero when any thruster is on. The vehicle will therefore not spin-up perfectly about

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*The specification of an angular acceleration of 0.5 RPM/minute to achieve a final 4 RPM rate in 8 minutes is for this study only, and is arbitrary. The actual spin up profile will depend on experiment requirements.

$\underline{U}_{MAX}$ and components of angular velocity will develop about the other axes. This condition is commonly known as wobble, herein defined as motion of the angular velocity or spin vector with respect to the axis of maximum moment of inertia.

2.3 Simulation

The rigid body dynamics of Skylab B are simulated by integrating the Euler equations of rotational motion on a digital computer. The effects of the pulsed-thruster spin-up procedure are studied for the case where the location of the inertia axes is known precisely. The effects of error in the knowledge of the location are investigated by using the same thruster firing profile to spin a vehicle whose inertia axes are rotated by amounts δ.

The off-nominal motion, that is, the wobble, is described by the behavior of a unit angular velocity or spin vector, $\underline{U}_S$, in a body-fixed coordinate system coincident with the principal axes. The wobble angle, Θ, is defined as the angle between $\underline{U}_S$ and $\underline{U}_{MAX}$.

3.0 RESULTS

In the first case analyzed, the location of $\underline{U}_{MAX}$ is known precisely ($\delta = 0$). Figure 3 shows $\underline{U}_S$ projected onto a plane perpendicular to $\underline{U}_{MAX}$ during the first part of the spin-up procedure. At $t = 0^+$ (the spin vector does not exist at $t = 0$) Θ is large but the spin rate is very small. At the end of the first period (ten seconds), Θ is very small. After a firing period, $\underline{U}_S$ always returns to approximately the same position it had at the beginning of the period. The spin rate increases and the maximum deviation in Θ during the period decreases in each successive period. Figure 4 shows the locus of the positions of $\underline{U}_S$ at the end of each period for $\delta = 0$. The path of $\underline{U}_S$ for the torque-free motion after spin-up is an ellipse that is traced every 38 seconds. The shape of this ellipse is determined by the inertia properties and the size of the ellipse is determined by the location of $\underline{U}_S$ at t = 8 minutes. While tracing this ellipse, the wobble angle Θ, which is caused by the spin-up procedure, varies from 0.1 to 0.2 degrees on the axes of minimum and intermediate principal moments of inertia, respectively, as shown in Figure 4.

Figures 5 and 6 show the effect of errors in knowledge of the location of $\underline{U}_{MAX}$ using the same pulsed thruster spin-up procedure. Figure 5 shows the path of $\underline{U}_S$ (at the end of each thrust period) during spin-up and the final elliptical path for error angles, δ, of plus and minus 5° in $\underline{U}_{MAX}$ about $\underline{U}_{MIN}$. The zero error case is repeated from Figure 4. The responses for $\delta = 5°$ and $\delta = -5°$ during spin-up are dissimilar because of the pulsed thruster spin-up procedure. The elliptical paths differ in size since the locations of $\underline{U}_S$ at t = 8 minutes are different. When an idealized constant moment is applied instead of the pulsed thruster procedure, the response is identical in shape for either positive or negative δ, though the positions of $\underline{U}_S$ are rotated 180° in the principal plane. The path during spin-up with idealized moment is quite similar to the pulsed spin-up paths of $\underline{U}_S$ for $\delta = 5°, -5°$. Figure 6 shows the behavior of $\underline{U}_S$ during spin-up and the final elliptical path for errors $\delta = 0, \pm 0.5°$ in $\underline{U}_{MAX}$ about $\underline{U}_{INT}$.

Limits on the wobble angle θ as a function of spin rate are derived in Reference 3 and are shown in Figure 7. Both limits were derived from published data on the ability of the inner ear to sense angular acceleration. The lower limit is based on threshold data on the blurring of vision and the higher limit on the threshold of the Coriolis illusion in a rotating environment. Also shown are envelopes of the maximum values of θ generated during the pulsed thruster spin-up procedure for $\delta = 0, \pm 0.5°$ about $\underline{U}_{INT}$, and $\pm 5°$ about $\underline{U}_{MIN}$. For $\delta = 0°$, the maximum wobble angle is always at least one order of magnitude smaller than the lower limit. With error in the location of $\underline{U}_{MAX}$ the maximum wobble angle is at most 15% the value of the lower limit. It should be noted, too, that no wobble damping or active vehicle control have been included in the simulation.

In any elastic body undergoing periodic loading there is always a certain amount of damping caused by the hysteresis effects of stress cycles. In a torque-free environment, this damping will eventually eliminate all wobble of a spinning body. Then $\underline{U}_S$ and $\underline{U}_{MAX}$ will coincide with $\underline{U}_H$, a unit vector in the direction of the angular momentum vector. In a torque-free environment, the angular momentum is constant in size and orientation in an inertial reference frame. Thus, in the absence of external torques, $\underline{U}_H$ at the end of spin-up is the ultimate location of $\underline{U}_S$ with respect to inertial space.

Figure 8 shows the path of the unit angular momentum vector $\underline{U}_H$ during spin-up and also its ultimate location at the end of spin-up for $\delta = 0, \pm 5°$ about $\underline{U}_{MIN}$. If the assumed axis of maximum moment of inertia is initially aligned with the solar vector, whether or not there is an error in the knowledge of its location, the reference frame is a solar inertial coordinate system with the solar vector out of the page at the origin. The ultimate locations of $\underline{U}_H$ are nearly identical and less than two degrees from the solar vector. The differences in behavior in these three cases arise from the manner in which $\underline{U}_H$ spirals in to its ultimate location. Figure 9 shows the behavior of $\underline{U}_H$ during and after spin-up for $\delta = 0, \pm 0.5°$ about $\underline{U}_{INT}$.

The solar power supply decreases with an increase in the angle between the Skylab Z-axis (solar array normal) and the solar vector. It is approximately proportional to the cosine of this angle. An error angle $\delta = +5°$ about $\underline{U}_{MIN}$ results in the largest angle between $\underline{U}_{MAX}$ and $\underline{U}_Z$, a unit vector along the Z-axis. This angle, which is 7.2°, is the third side of a spherical triangle with the other sides being $\delta = 5°$ and the 4° angle between $\underline{U}_{MAX}$ and $\underline{U}_Z$. Since the ultimate motion in a torque-free environment will be constant rotation about $\underline{U}_{MAX}$, which will be colinear with $\underline{U}_H$ and $\underline{U}_S$, $\underline{U}_Z$ will ultimately trace a cone of 7.2° half-angle about $\underline{U}_H$ in an inertial reference. The power penalty will then be less than 1%. Figure 10 shows the coning path of $\underline{U}_Z$ as a circle centered about the ultimate location of $\underline{U}_H$ in a solar inertial reference frame. The dashed line in Figure 10 shows the path of $\underline{U}_Z$ immediately after spin-up, and therefore includes the effects of the wobble induced by the spin-up procedure.

4.0 CONCLUSIONS

The open-loop pulsed thruster spin-up procedure using three SM RCS thrusters introduces very little wobble into the rigid body motion of the spinning Skylab. An error in the knowledge of the location of the axis of maximum principal moment of inertia causes the wobble to increase, but it still remains small with respect to the anticipated physiological detection threshold.

The sequence used to pulse the thrusters causes a small angular deviation between the angular momentum vector and the solar vector. This deviation can be decreased by optimizing the thruster firing sequence. In any event, solar power loss is less than 1%, which is certainly acceptable.

L. E. Voelker

1022-LEV-cf L. E. Voelker

Attachments
References
Figures 1-10

REFERENCES

1. Voelker, L. E., "Influence of Vehicle Dynamics on the Artificial Gravity Experiment on the Second Saturn Workshop," Bellcomm Technical Memorandum TM 70-1022-5, Washington, D.C., April 1970.

2. Hough, W. W. and Nelson, L. D., "Minimization of SM RCS Fuel for Skylab Attitude Maneuvers," Bellcomm Technical Memorandum TM 70-1022-13, Washington, D.C., August 1970.

3. Ravera, R. J., "Physiological Limits on Skylab B Wobble During an Artificial Gravity Experiment," Bellcomm Memorandum for File, Washington, D.C., September 30, 1970.

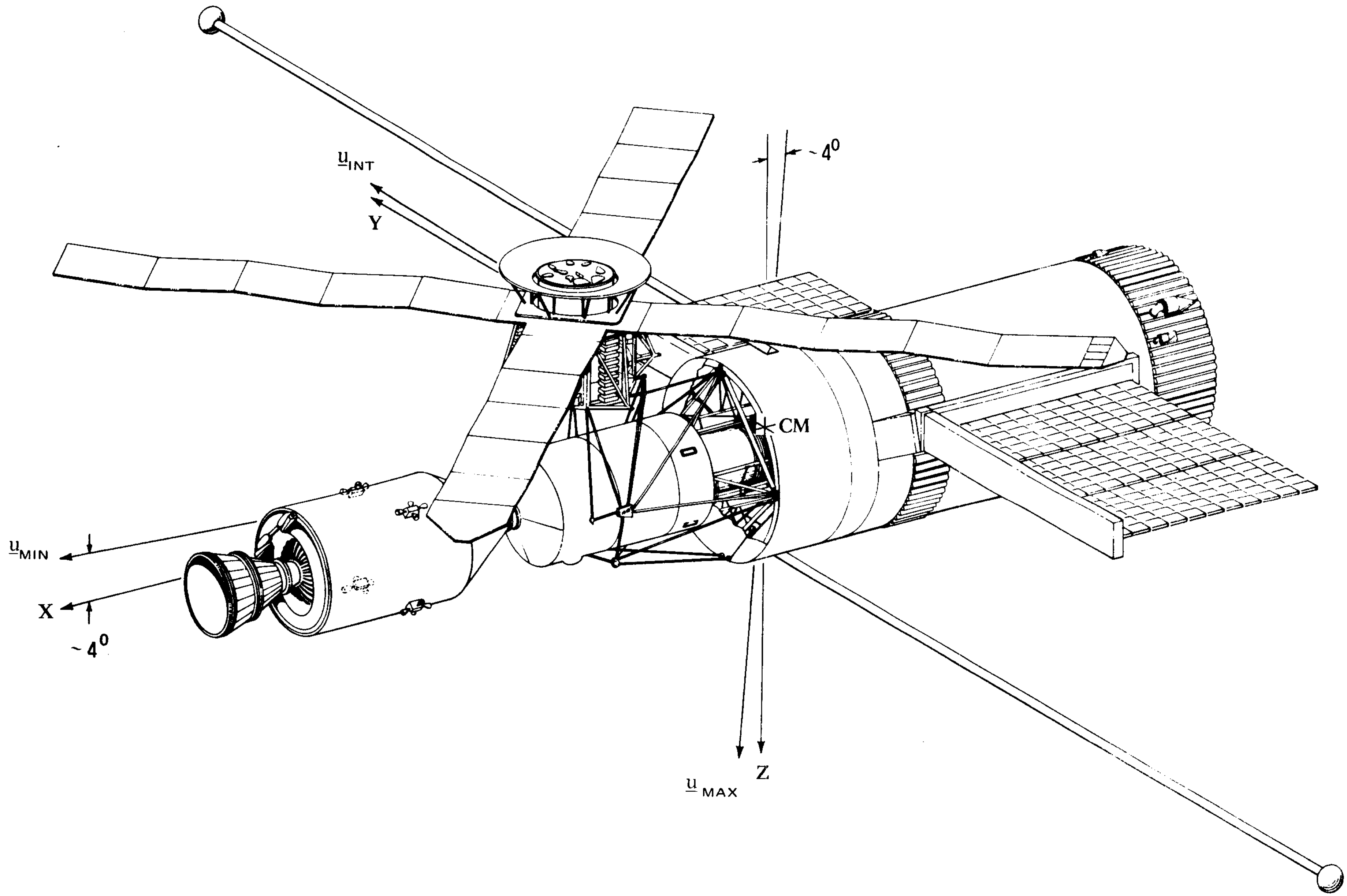


FIGURE 1 - SKYLAB BALANCED FOR ARTIFICIAL GRAVITY EXPERIMENT

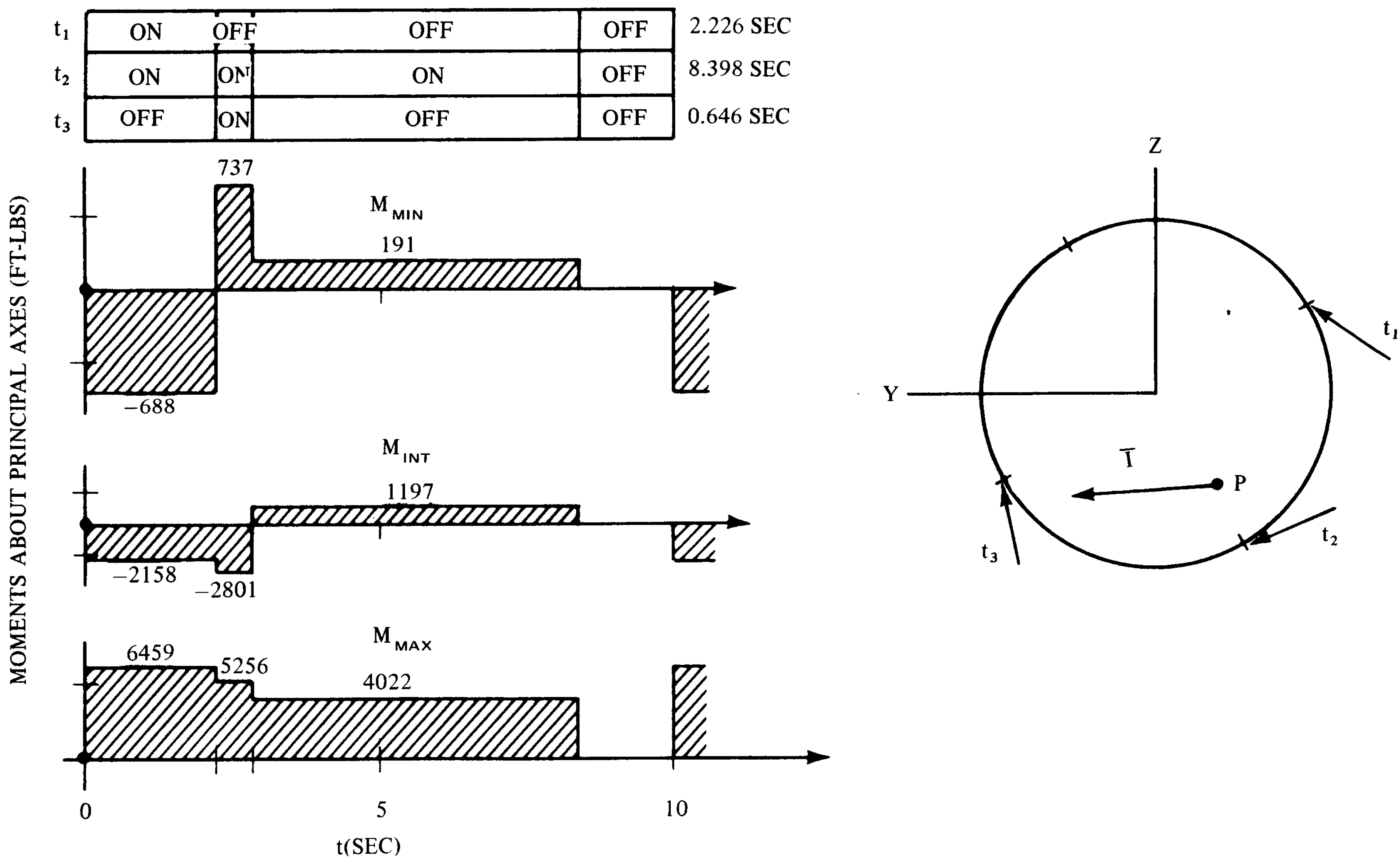


FIGURE 2 - PULSED ROCKET SPIN-UP PROFILE

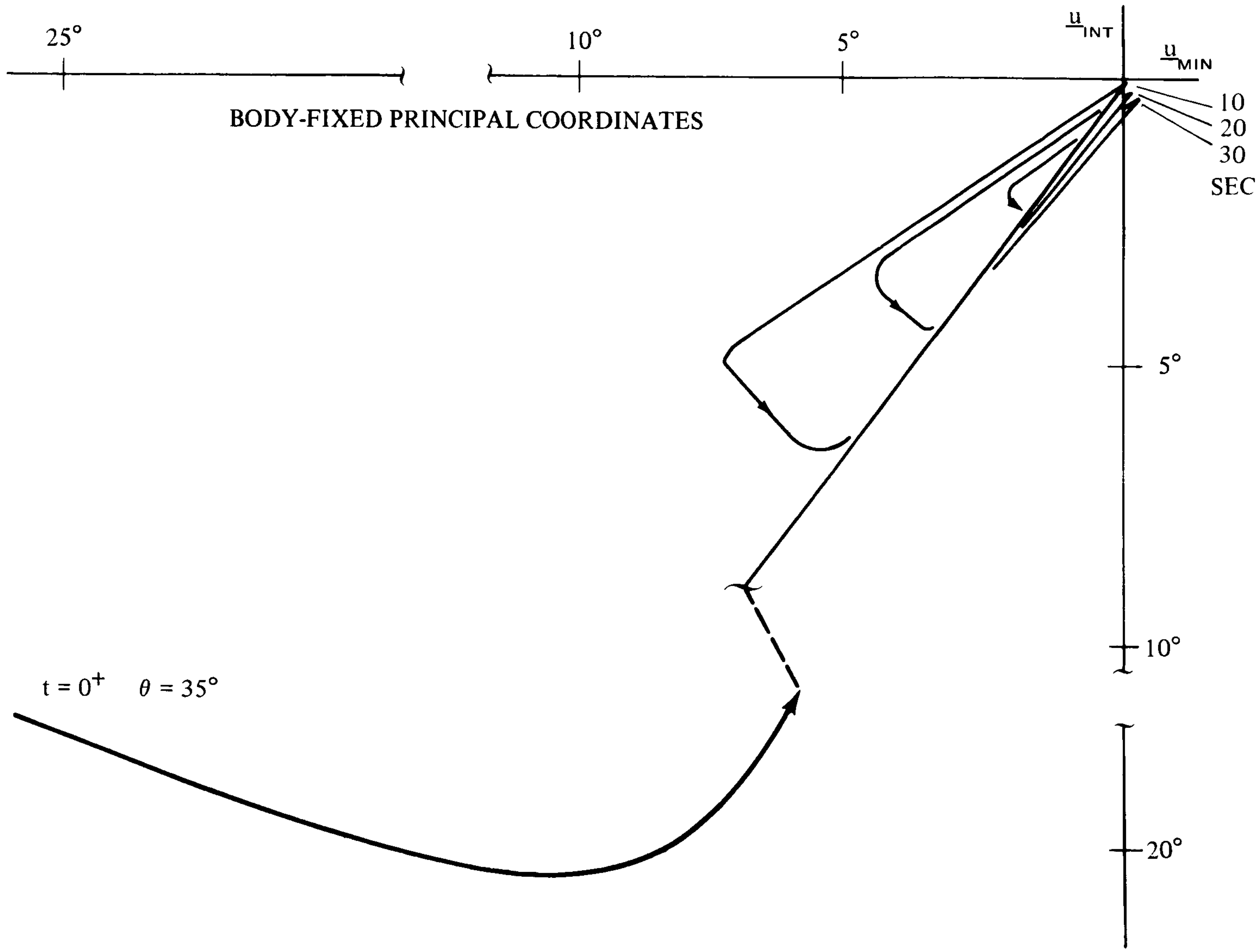


FIGURE 3 - PROJECTION OF UNIT SPIN VECTOR $\underline{u}_s$ ONTO PRINCIPAL PLANE DURING SPIN-UP

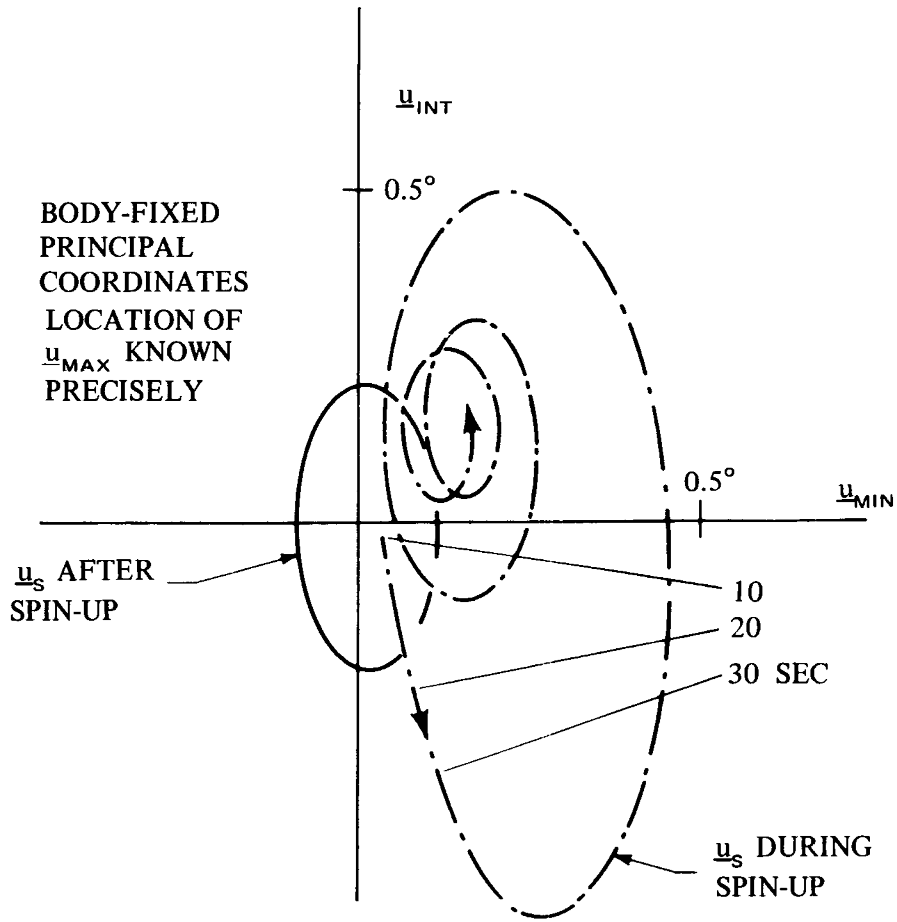


FIGURE 4 - PROJECTION OF UNIT SPIN VECTOR $\underline{u}_S$ ONTO PRINCIPAL PLANE DURING AND AFTER SPIN-UP

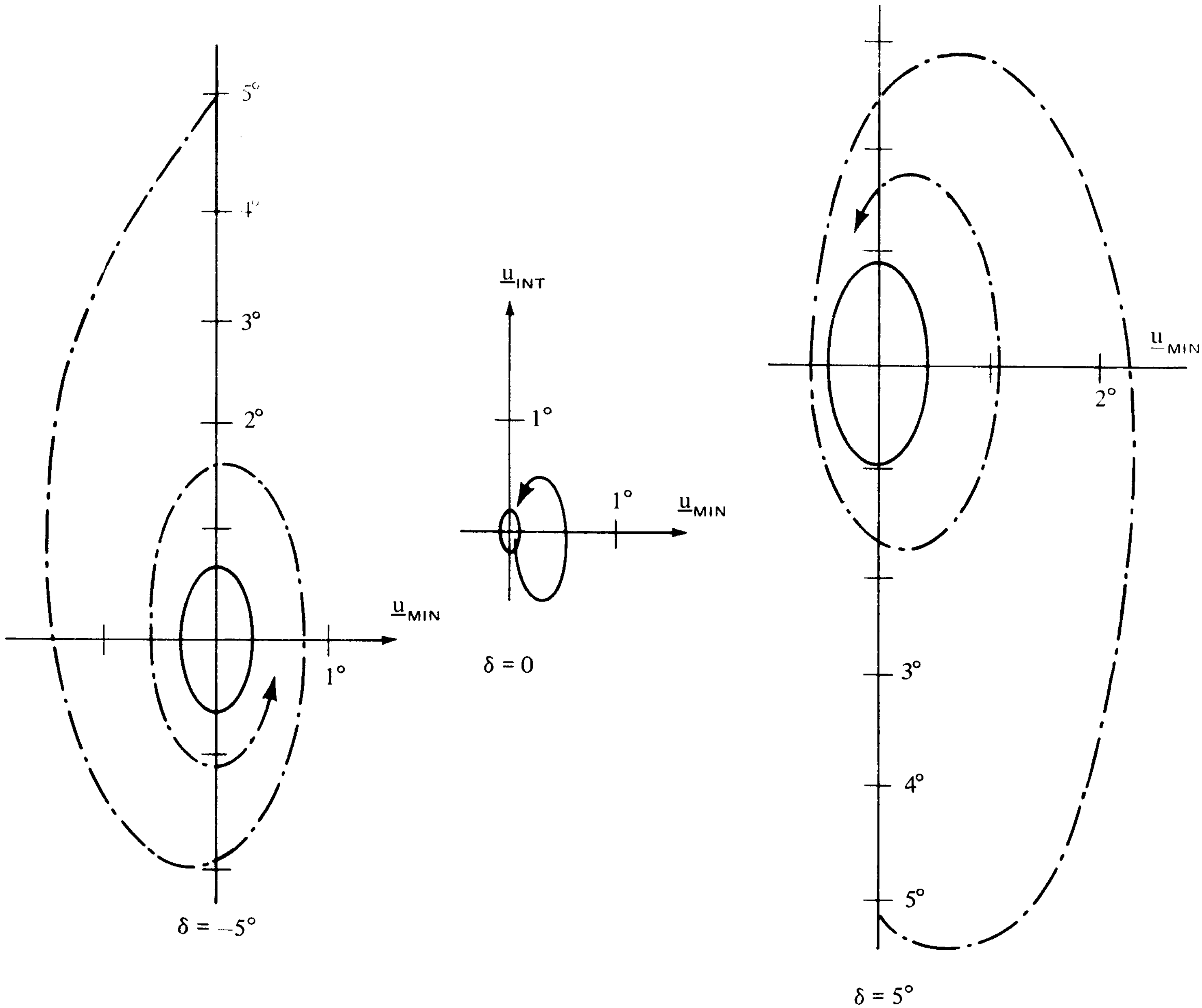


FIGURE 5 - PROJECTION OF UNIT SPIN VECTOR $\underline{u}_S$ ON TO PRINCIPAL PLANE FOR ERROR δ IN LOCATION OF $\underline{u}_{MAX}$ ABOUT $\underline{u}_{MIN}$

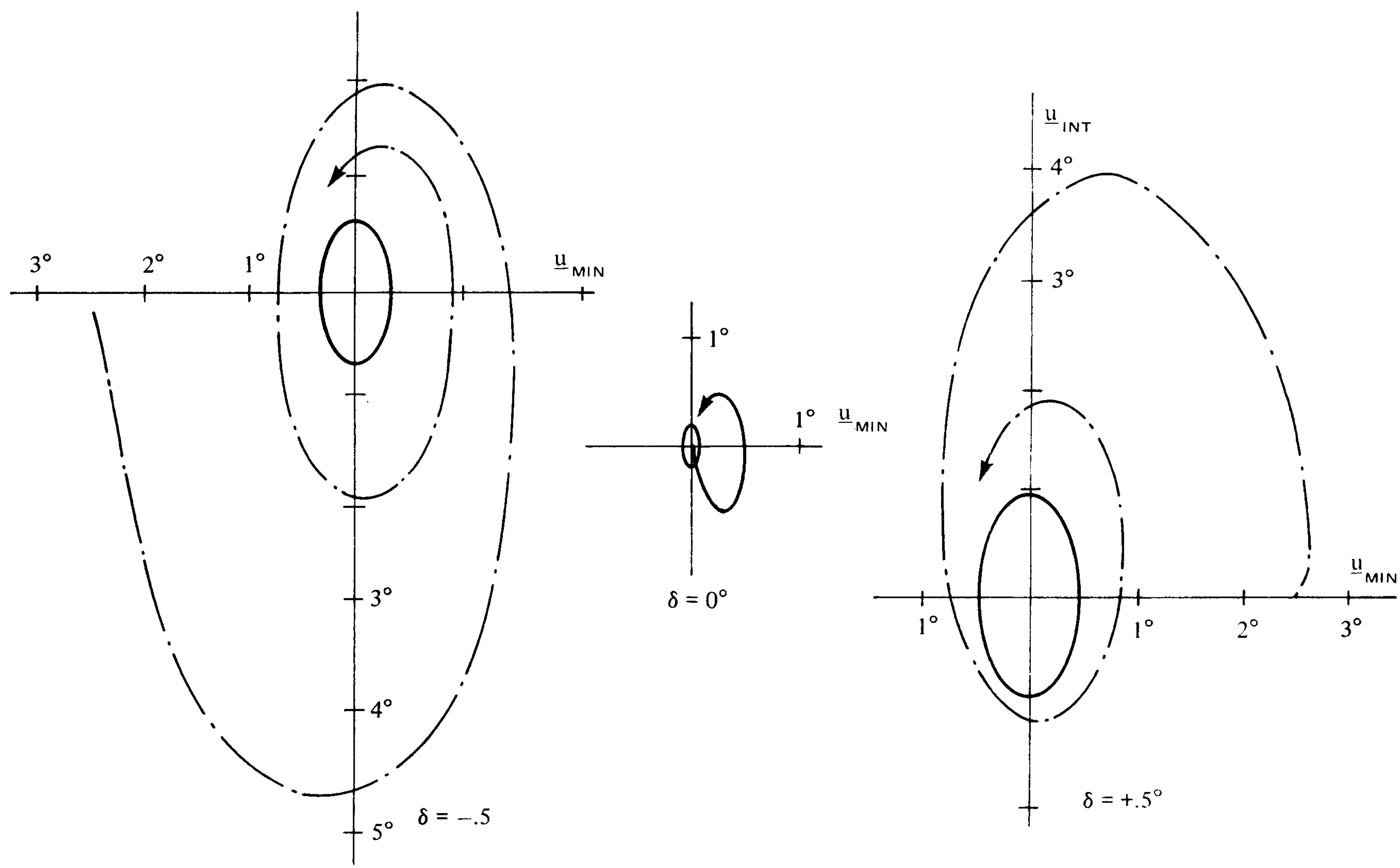


FIGURE 6 - PROJECTION OF UNIT SPIN VECTOR $\underline{u}_S$ ON TO PRINCIPAL PLANE FOR ERROR δ IN LOCATION OF $\underline{u}_{MAX}$ ABOUT $\underline{u}_{INT}$

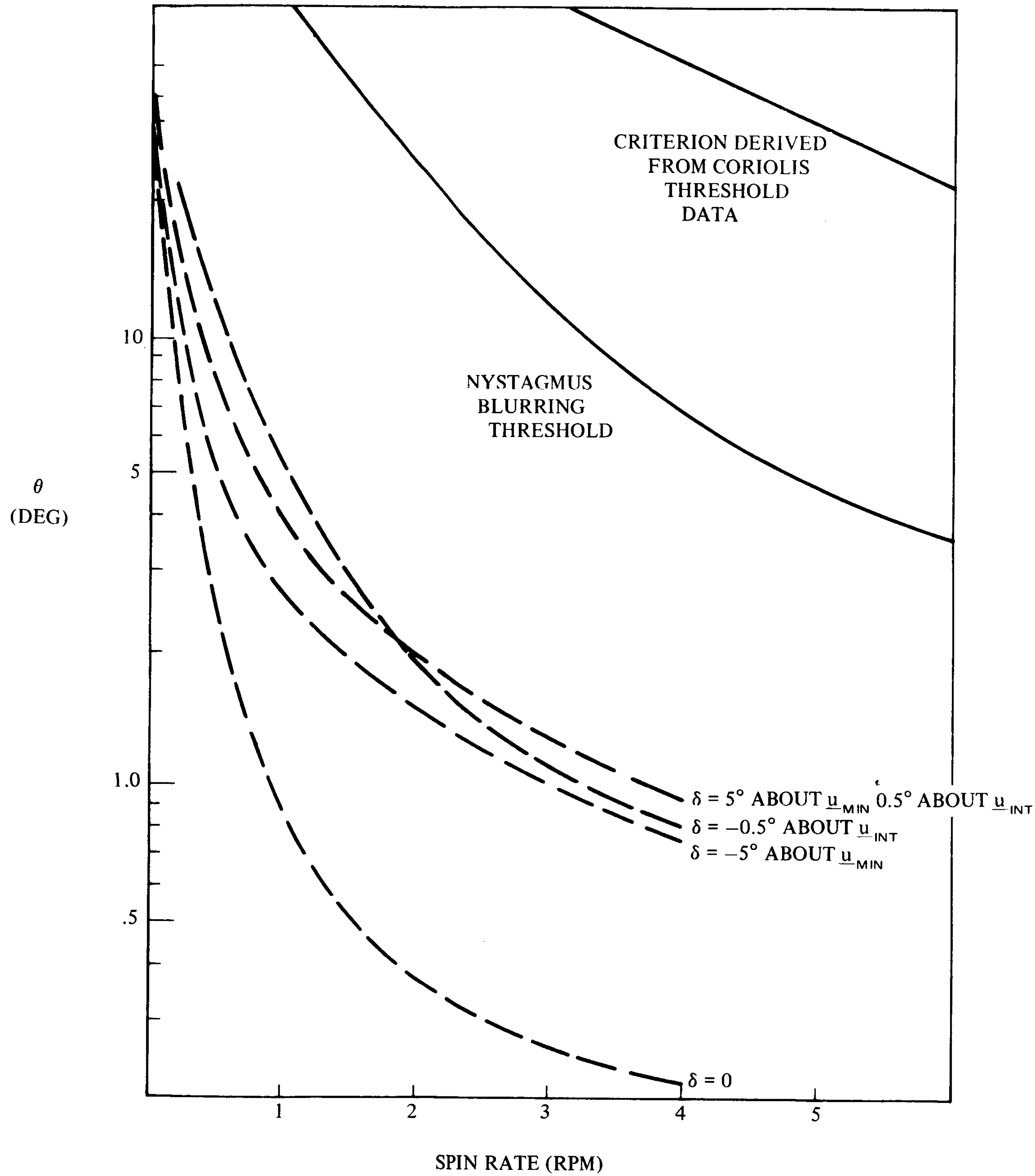
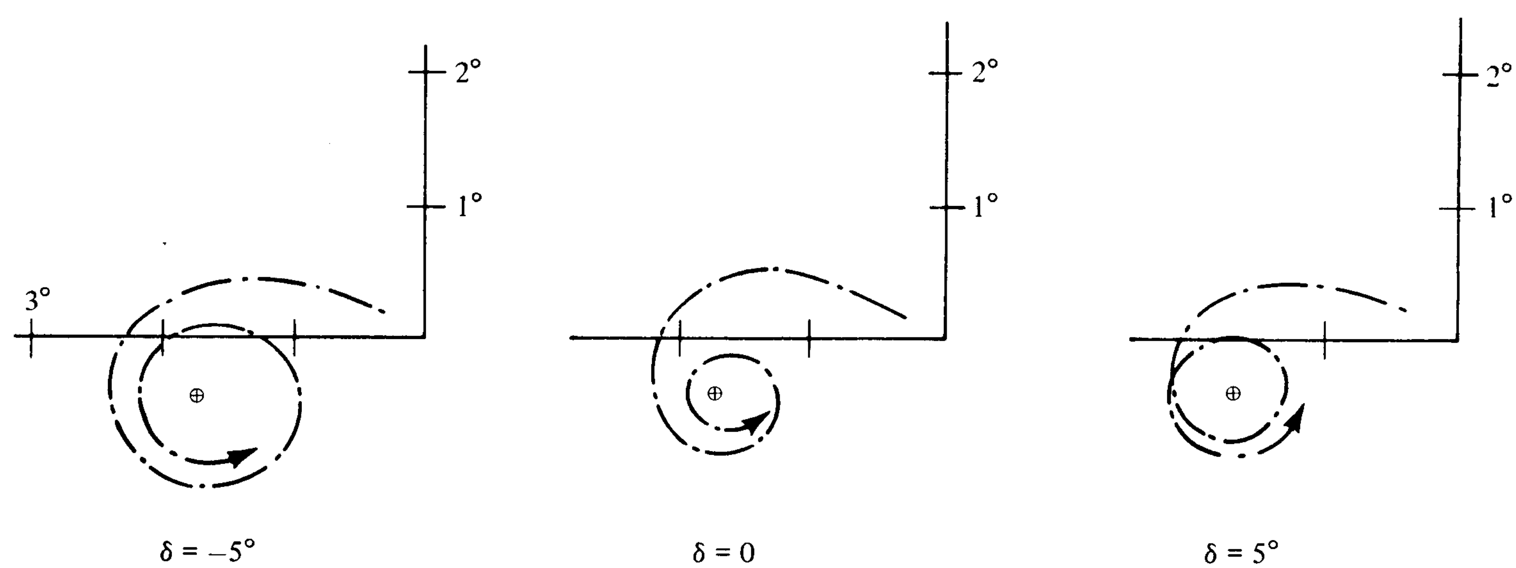


FIGURE 7 - MAXIMUM WOBBLE ANGLE VS SPIN RATE FOR FIVE CASES OF ERROR, δ, IN KNOWLEDGE OF LOCATION OF $\underline{u}_{MAX}$

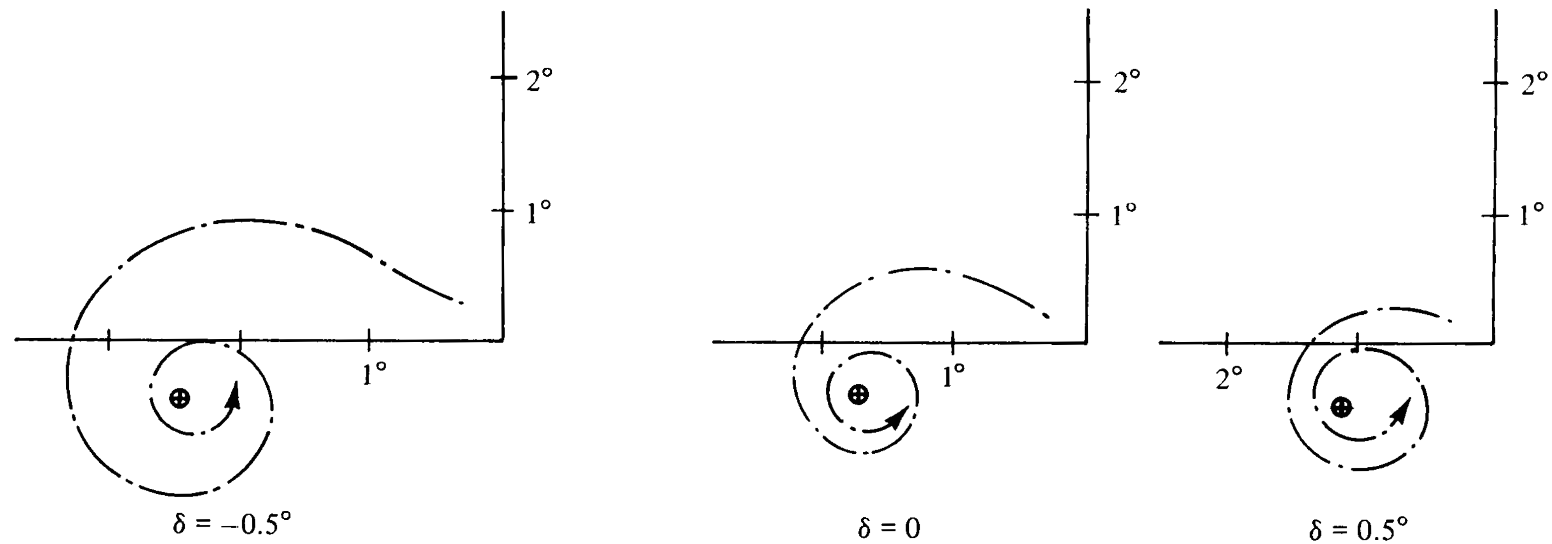
SOLAR VECTOR OUT OF THE ORIGIN



⊕ LOCATION OF $\underline{u}_H$ AFTER SPIN-UP

FIGURE 8 - UNIT ANGULAR MOMENTUM VECTOR $\underline{u}_H$ IN SOLAR INERTIAL COORDINATES DURING AND AFTER SPIN-UP FOR ERROR δ IN LOCATION OF $\underline{u}_{MAX}$ ABOUT $\underline{u}_{MIN}$

SOLAR VECTOR OUT OF THE ORIGIN



⊕ LOCATION OF $\underline{U}_H$ AFTER SPIN-UP

FIGURE 9 - UNIT ANGULAR MOMENTUM VECTOR $\underline{u}_H$ IN SOLAR INERTIAL COORDINATES DURING AND AFTER SPIN-UP FOR ERROR δ IN LOCATION OF $\underline{u}_{MAX}$ ABOUT $\underline{u}_{INT}$

SOLAR INERTIAL COORDINATES

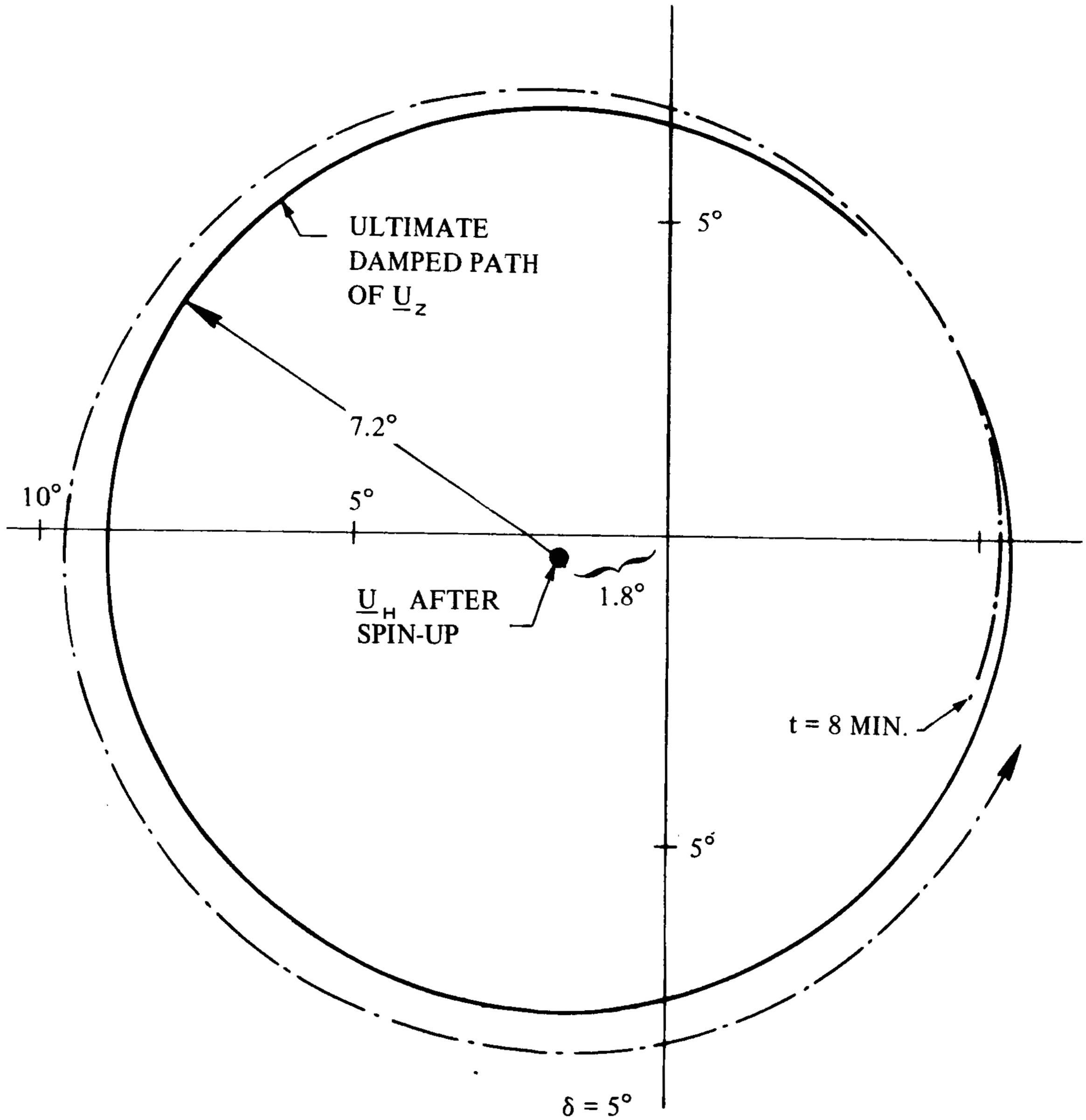


FIGURE 10 - SOLAR ARRAY UNIT NORMAL VECTOR $\underline{u}_Z$ AFTER SPIN-UP WITH 5° ERROR IN LOCATION OF $\underline{u}_{MAX}$ ABOUT $\underline{u}_{MIN}$